MUSICOMETRY OF INDIAN MELODY

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To-day when the rapid progress of science and technology is bringing the east and the west closer and closer day by day in various fields of artistic appreciation and creativity and when, in the field of music, the west after its wide and weary excursions in atonality has started looking back to melody as wistfully as the east in its search for new and exciting tone-colours is being drawn towards polyphony, the system of Indian melody, if it is to become again what it had been at its best and is to seek self-fulfilment through self-perfection, must try to know itself — its own purpose and character, content and form — more clearly and precisely than it happens to do at present. In other words, it must begin to realise more seriously and intelligently in the present age of realistic and rational thinking, that being what it is, a system of intense and unalloyed melody, it has to find out and obey certain fundamental and well-defined laws of logical structure in view of the fact that its "..form, proportions, melodic outlines, accents, rhythms and so on, are subject to a few rather simple axioms to be formulated in a new theory of melody, still waiting for its convincing and comprehensive explanations .."[1]

That the primeval Indian system of melody had an elaborate logical structure based on certain definite axioms may be reasonably deduced, if not conclusively proved, from the immense importance attached in the Vedic ages to tonal purity and precision in the chanting of Vedic hymns, and also from the meticulous care then taken to ensure a flawless and faithful transmission of the knowledge and ability of tonal theory and practice of Vedic chants from generation to generation.

Accepting the indisputable fact that Vedic hymns have always been held in highest reverence in India for their deep spiritual meanings and have inspired Indian philosophy, literature and art as nothing else has done, one can hardly explain the uncommon insistence constantly laid upon the tonal accuracy of their chants except by postulating that melodic phrases were deemed to be, and probably were, as important as verbal phrases for the revelation of those meanings and for the dawn of the inspirations that were supposed thereupon to follow. This is the only explanation capable of accounting rationally for the *Sama Veda*, the Veda of musically composed hymns, out of all the Vedas having been identified by the Divine in Krishna with Himself in the *Geeta*[2], suggests that the system of melody employed in Vedic chants had developed a technique through which words and their intonations could be welded together to constitute semantically an indivisible whole for communications of spiritual meanings and births of creative inspirations, which would otherwise remain uncommunicated and unborn. A technique such as this, assuming that it was real, would be simply unthinkable without a logical structure and an axiomatic base. But since only disjointed and hazy traces of it are available now here and there in theory and practice, one can only wonder if it would be possible by any modern means to re-discover or re-construct its structure and base that were once living realities.

The transition from the Vedic to the post-Vedic form of Indian melody, though marked by an emphasis on wider sensuous appeal and accessability, tended unmistakably to stain its spiritual preferences and predilections. Evolved in the succeeding ages of growing cultural mixtures in such a manner as to be able to attract spiritualists and sensualists alike, melody was pursued on the academic plane as *Margi Sangeet* the music of search, and was developed into an organisation of sound which claimed, in its perceived vibrational details, to enchant the human mind on the highest level of its awareness of abstract beauty and, in its conceivable non-vibrational essence, to secure mind's emanicipation from existential thought-process by concentrating it on blissful abstractions of subtle ideas and intimate experience.3 The technique of this organisation sought to make melody a precise and powerful vehicle of expressions and symbolisations of all varieties of ideas and emotions and tried to provide it with an axis of an underlying orderliness together with levels of a self-generating dynamism for its sweeping movements from heights of objective creativity to depths of subjective receptivity, making it as eloquent in its audible appeal as in its silent connotations. It appears from this that the secret of the technique of *Margi Sangeet*, if it really was what it claimed to be, lay in certain facts of dynamic orderliness inhering the phenomena of melody, by virtue of which patterns of orderliness got transformed into images of beauty and currents of dynamism turned into waves of emotions and ideations enveloping man's heart and soul alike.

In the light of the history of Indian culture in the post-Vedic ages, when philosophy and religion joined hands to expound and extol the twin cults of *Yoga* and *Bhakti* through the manifold media of art and literature, one is irresistibly led to suppose that the system of Indian melody came to function on its higher levels as an aesthetic channel for spiritual pursuits of self-realisation and self-expansion. Since the former, according to tenets of *Yoga*, required a focal point of concentration and the latter, according to principles of *Bhakti*, pre-supposed a wide range of meditation, melody was provided with a base of self-generating dynamism and a structure of self-balancing orderliness. While the base symbolised the individual in man through the singularity of the tonic note, (the *Shadja*), with all its possibilities of creating and being re-created in a melodic pattern, the structure expressed the universal in him through the totality of the inter-relations sonance, assonance, consonance and dissonance.3a Built up thus, melody became at once a field and a force for the establishment of rapport and harmony between the personal and the cosmic selves of man for his discovery of and attunement to the infinite and infinitesimal truths of Unity in Diversity, symbolising through the ethereal language of tonality the mystic imagery of fleeting series of tension and relief through the hide-and-seek of discords and concords between the central One and the orbital Many, each of them being immanent in and emergent from the other.

This system of melody, as is evident from all that remains of it today, was structurally a balanced combination of fixity of artistic norms and fluidity of artistic freedom, having its base in an artistic complex of many intertwined balancings of various pairs of opposites, like consonance and dissonance, symmetry and asymmetry, definiteness and infinity. In fact, it was these balancings from top to bottom which cumulatively infused meaningfulness into melody. But every act of balancing pre-supposes certain entities having positive and negative values in a certain context, all such values being measurable in some form or another in time or space. It follows, therefore, that the Indian system of melody, a composite whole of many balanced constituents, did not only possess measurability of contents, but demanded of each one of its contents to be in proper measure in order to be able to contribute to the totality of its ultimate meaningfulness. In other words, the system did not only have a mathematical character but required its character to be preserved in a state of correct balance for the emergency of its psychological qualities.

Now, how the twofold meaningfulness of music, absolute and referential, revealing peculiarities of melodic structure, on the one hand, and trends of emotional and ideational expressions and symbolisms, on the other, could emerge from an integral process of measured balancings of musical contents, and how facts of mathematical proportions came to be followed by phenomena of psychological discernibility, has been and shall ever remain a mystery to human intelligence. But that the world of music is in the ultimate analysis a fabric of numbers, and melody a sound-manifestation of an ordered dynamism, the source of which lies in numeral relations, is a truth from which it is difficult for any logical mind to escape. That the conscious mental process of creation and appreciation of melody is initiated by, and finds its fulfilment in, the sub-conscious mental tendency of grouping for, and reposing in, dynamic orderliness born of numeral relativity, appears to be the only explanation of this mystery and implication of this truth. Accepting this to be so, one may credit the Indian system of melody with an adventure of establishing a positive line of communion between the conscious and the unconscious centres of the mind for achieving mental integration through balanced patterns of numeral relations. To understand, therefore, the structural logic and the basic axioms of this system, partly lost due to historical causes and partly reduced to confusion on account of over-all cultural decadence in the post-epic era, one can hardly find a more promising avenue of investigation than to reason out how numeral relations entered into their making.

The task, by no means easy, is rendered more difficult by the appearance of a large area of vacuum in musical rationality in the musicological literature handed down to us in writing from, roughly, the age of Buddha (500 B.C.). The comments and treatises on Indian musicology, from the *Shiksha-Grantha*-s down to the *Natya Shastra* (200 B.C. — 200 A.D.) and the *Sangeet Ratnakar*, (1210 — 1247 A.D.), apart from mixing up the physics and the metaphysics of music, have preferred to define musical ideas and terms more from a psychological, than a mathematical angle. This has made an already obscure subject obscurer, bringing the added misfortune that a subjective approach to realities, more accessible objectively, has continued to be a dominant trend in Indian musicology.

A typical example of this practice may be cited from Bharata's initial definition of notes (*swara*-s) as sonant (*vadin*), consonant (*samvadin*), assonant (*anuvadin*) and dissonant (*vivadin*).[4] Now, all the above four qualities of notes have subjective rather than objective nomenclatures, indicating how they are perceived psychologically without any reference to how they may be conceived mathematically. A certain pitch of sound becomes a musical note only when it stands in a certain mathematical relation, vis-a-vis the tonic note, and when the relationship is of an accepted category, from the point of view of its microtonal locaton (*srut-sthana*) in the scale. To define the note psychologically, is to describe its musical effect, while to define it mathematically, is to state its cause. In the realm of musical art it may be necessary to know the tonal effect of a note, but in that of musical science it is imperative to know its tonal cause.

The situation hardly improves by Bharata's famous employment of the process of *chatussarana* for finding out the twenty-two *sruti*-s of the musical scale on two separate *veena*-s having the same order of tuning.[5] This process, not a little tortuous, presumes both the *veena*-s to have been tuned in the scale designated by him as that of the *Shadja-grama*,[6] the only objective description of the scale being the fact that the series of its seven notes is punctuated by intervals of 3, 2, 4, 4, 3, 2 and 4 *sruti*-s respectively.[7] Now, since the term *sruti* has nowhere been defined by Bharata or his predecessors as a unit of tonal measurement, and since the Indian musical scale is certainly not one of equal temperament, the *Shadja-grama* remains, for all scientific purposes, uncovered by this definition and objectively vague. So remains the scale of the *Madhyama Grama* which has its fifth note (*Panchama*) lower than that of the *Shadja-Grama* by just one *Pramana-sruti*,[8] the latter term also remaining objectively vague. Modern musicologists have equated Bharata's *Pramana sruti* with the *Pythogorean Comma* which is equal to 81/80.* They have also tried to find out the mathematical values of the other *sruti*-s referred to by Bharata. But what has been or is being done now is, in a way, a belated realisation of the fact that the trend of subjectivising concepts that needed objective elucidation has been the main cause for the obsolescence of the ancient musicological literature.

This trend has been sought to be explained by medieval musicologists on the ground that "discernment of tonal values is possible only by the sense of consonance."[9] But this is hardly a valid explanation, because inferences drawn by subjective faculties about objective phenomena which, while having psychic effects have physical causes, may have an artistic value but not a scientific validity. True, tonal qualities are not acoustical but mental phenomena and, as such, "they depend for their definition upon the psychological laws governing human perception, in the context in which the perception arises, and upon the learned response patterns which are part of the context."[10] But since the 'perceptions' and 'learned response patterns' are, in the very nature of things, subject to fluctuations from time to time and to variations from individual to individual, and also because 'context' tends to differ from situation to situation, musicology must employ mathematical terms verifiable by physical means while defining musical ideas and terms in order to make its definitions more lucid, precise and open to rational understanding.

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*While 81/80 (the difference between a major tone and a minor) is generally equated to a Pramana sruti, its Western equivalent is the comma of Didymus and not the comma of Pythagorous (=531441 : 524288) which is the difference between twelve fifths and seven octaves. Vide *Grove's Dictionary of Music and Musicians*, p. 523 vol. IV (Macmillan, 1954).—B.C.D.

It is true, as Victor Zuckerkandl puts it, that "music is .. a miracle, knowledge can never do more than touch its surface", still as he further points out, "between the known surface of music and the central mystery there lies a vast area of potential knowledge as yet largely unexplored, a great challenge to thought a promise of high reward".[11] That the ancient Indian adventure in melody was fully conscious of this 'miracle' is suggested by its equation of the delight born of music with that accuring from God-realisation — *brahmanand-sahodarah.* But that it tried nevertheless to explore the mystery of the miracle by proceeding from its vibrational surface of pleasantness — *rakti* — to its non-vibrational centre of emancipativeness — *mukti* — is also borne out by its belief that "God dwells where His devotees sing."[12] It may, therefore, be worth-while to see how far an objective view of melody can elucidate its subjective experiences, and how far a survey of its outer facts can clarify its inner meanings.

Melody defined in modern times as a "horizontal resolution of harmony"[13] by Helmholtz, has been described further as "a succession of single tones of varying pitches,"[14] "making a design in pitch and rhythm to produce a satisfying musical effect",[15] in which "motive, phrase, and period are formed by the combination in proper relationship of movement and sound."[16]

Though the motivation of melody resulting from "some inner urge of higher aesthetic synthesis"[17] has always been found to be difficult of analysis, its tonal dynamics and architectonics have been sought to be analysed in modern times from various angles. In one such analysis melody has been described as "the fundamental consciousness of progression from point to point, with the rhythmic melodic outline as the essential agency that binds the whole together into a coherent self-supporting entity".[18] In another analysis it has been defined as "a sequence of tones .. when it is apprehended in terms of unified and single response .. which arises not from the tones themselves, but is contributed by the act of the listener".[19] This is so because "melody must be regarded as a form of thought as well as a sound-phenomenon," there being in it "the unity of a single sentence, which is a logical relation".[20] It is this unity that makes melody "an organism or design, initiated in a local formulation of ideas and evolved through their projection into the larger pattern on the whole, in faithful realization of an expressive intent."[21]

The expressive intent of melody differs from that of words, in that "the function of words is to express the concept of actuality," while "the function of melody is to express the structural scheme of actuality. Words generate concepts which may or may not stimulate feelings. Melody, on the contrary stimulates feelings..which may or may not generate concepts".[22] This expressive character of melody makes a tune "a sentence" that "makes sense".[23] It also gives it the semblance of an 'idea', by virtue of which "a unique response of thought and mood is awakened in the listener".[24]

The character of melody, as described above, is possible only as a result of its tonal dynamism. As Edmund Gurney puts it "each tone contains an inherent yearning to move on to some other tone .. yielding a form, a unity to which all the parts are necessary in their respective places".[25] Describing the tonal movement in a melody Max Schoen says, "each tone commands what tone shall follow it, each interval commands another interval to succeed it, the whole movement pulling to one, and only one close."[26]

Such intimate relations between tones and their intervals argue the presence of an energy-function in melody, which Schillinger describes as an interaction between the forces of impetus and inertia. He says, "the projection of a melody is a mechanical trajectory. Its kinetic components are balance, impetus and inertia."[27] This trajectory according to John Dewey produces a spell of tension and relief, out of which "music weaves its web".[28] And perhaps this is what makes melody "the idealisation of the natural language of passion"[29], the realisation of which has made Bergson say, "our personality is precisely that: the continuous melody of our inner life. "[30] Another view of melody likening it to an "image of energy" is presented by Jule Cambriew, which symbolises an intense struggle ending in a serene repose.[31] Since every known form of energy has its generating centre, the above likeness can be completed only by accepting the tonic note of the melody as that centre. As H. Parker puts it, "this (tonic) note .. becomes a plan or a purpose, .. it leaves in the mind a memory of itself .. an expectation and desire to hear the note again .. and when the tonic eventually triumphs, it fulfils not only itself, but all lesser desires of the melody .. for in hearing the final note we hear the memories of all previous notes also."[32] (see ft. note 3a).

Thus, in a melody that is ever tending with various degrees of urgency and of strain to its centre of gravity, the Zero axis of the tonic note represents an equilibrium of this tendency, expressed in terms of centrifugal and centripetal forces. The amplitude of a melody increased by the former causes tension and decreased by the latter brings relief. "Form in a melody, then, is a moving equilibrium between opposite energies, a progressive everchanging stability between the feelings of tension and relaxation."[33]

The Greek word 'tonos' means tension. Melody being a sequence of dynamically interrelated tones, has tension as its basic material and speaks primarily the language of tension. The totality of this tension is but the psychological reflection of the image of the integral order of the tonal movement, which projected by physical media creates context, meaning, structure, variety and unity in melody and ultimately melody itself. Between this image and its reflection, there stands the sum total of the inter-relationship of all the tones, which have the tonic note as their common dynamic centre, from which they all emerge and into which they all merge back. Their emergence holds a distinct promise of a dynamic experience and their merging back leaves behind a haunting memory of the occurrence of it.

A melody, as a tensional complex, strings its mutually related tones in a single file giving each one of them a certain degree of importance which determines the frequency of the appearance of each note in the total scheme of the tonal movement. According to Victor Zuckerkandl, "the movement proceeds on two planes simultaneously. In the background, as it were, the succession of main stations, in the foreground the detours .. by which station is linked to station." The function of the former is to provide the melody with a skeleton and of the latter to supply it with flesh and blood. "Beauty, individuality, distinction do not reside in the skeleton but in that which the skeleton supports .. the ear must instinctively be aware .. of the corresponding background-foreground relation of the melody. Since the beauty and interest and, in fact, the very meaning of every tune rests precisely in the relation between the flesh and blood of the musical foreground and the supporting skeleton of the background, a melody can be enjoyed only when the relation is understood."[34]

The question of relationship is ultimately connected with that of balance. A dissonance being formless tonal 'no' and a consonance a disembodied tonal 'aye' in melody, a melodic elaboration becomes an exercise in balancing the 'ayes' and 'noes' of tonality. As John Dewey puts it, a work of beauty is "one of the most delicately balanced organisms in the world. The balanced distribution of .. opposing energies provides the means which prevents variation from becoming confusion."[35] That a melody is a work of beauty *par excellence* is shown by the fact that it "holds together within itself the greater variety and scope of opposed elements", which, according to Dewey, is the "final measure of balance" of any artistic whole.[36] Since "balance is primarily a principle of rest" in which "all the elements of a work of art interact and counteract in such a way that an equilibrium of forces is maintained,"[37] a melody assumes the ultimate character of "a response in tension."[38] Of all the arts in which the principles of creating 'Unity of Complexity', and of putting 'variety in order' stand as central doctrines, the art of melody is the foremost. This probably is why Walter Pater has said that "all the arts strive to attain to the condition of music because in it the values of form and content are more completely blended than elsewhere."[39]

Balance in melody is achieved through tonal order, because "order is heaven's first law in art as in nature."[40] And tonal order is first and last mathematical order, tone being the real mediating factor between music and mathematics. A melody, in its last analysis, is, therefore, a tonal structure in which aesthetic unity in complexity is achieved by the subordination of all its tonal elements to the basic and dominating ground-ratio of the tonic. This is why the tonic has been called the *shadja* in Indian musical terminology which means that it is both the progeny and the progenitor of the other six notes of the diatonic scale, the initial cause as well as the ultimate effect of its musical meaning.

Gestalt psychology maintains that all our mental experiences occur in structural plans with inherent dynamic organization.[41] These patterns appear not only in the tone-to-tone motion, but equally in tonal groups, sub-groups, super-groups at several planes of a multi-tiered architecture bringing out the inherent symmetry of its design in the apparent asymmetricality of its details. As Ernest Mundt puts it, some variation must be introduced in order "to keep stimulation alive" and to achieve a "repetition veiled by complexity".[42] Asymmetrical proportion in a controlled deviation from symmetry, like a controlled excursion in dissonance within the defined bonds of consonance, imparts a "fitness to the design through the challenge of reconciliation between the regular and the irregular."[43] Consequently, "the disbalance of asymmetry in search of balance of symmetry is a powerful impulse in design" and to realise "a symmetrical relationship in an asymmetrical succession of elements" has been recognised as a triumph of art, more so of the art of melody.

The question of balance between symmetry and asymmetry, like that of balance between consonance and dissonance, has led in the West, as in India, to the related question of the numerical constituents of the tonal material, because a tone is but a psychological percept of the physical reality of a pitch, which in its turn is caused by a certain frequency of vibration in the air, a purely mathematical term.

According to Theodore Lipps, "each interval resolves towards that tone which has a vibration rate of some power of 2, 3 and 5."[44] "Only tones having vibrations of some power of 2, 3 and 5 have been considered capable of melodic relationship."[45] Max Mayer has suggested that vibration rate of 7 or one of its powers should be included in melody.[46] This shows the importance in melody of those harmonics which appear in the first three octaves, which is natural because a melody can be sung in not more than three octaves, and music, according to Curt Sachs, began with singing.[47] The prominence of the tonic triad is, in a way, a recognition of this importance. According to W.V.Dyke Bingham "the final tone of the melody must be one of the members of the tonic-triad in order to bring about feeling of finality."[48] This is supported by James Jeans who says that two tones sound well together when the ratios of their frequencies can be expressed by the use of small numbers, and the smaller the numbers the better is the consonance .. the farther we go from small numbers, the farther we go into the realm of discord. This was known to Pythagorus 2,500 years ago; he was the first, so far as we know, to ask the question, why is consonance associated with the ratios of small numbers ? And although many attempts have been made to answer it, the question is not fully answered yet."[49]

This question takes us into the deep waters of the 'why' and 'how' of the relations between music and mathematics, into which we are neither competent nor inclined to go. We can only quote in this connection the famous statement of Plato recorded by Plutarch that "God for ever geometrizes" and also of Galileo that "Nature's great book is written in mathematical language."

Turning to James Jeans, we find him saying that "a scientific study of the action of the universe has suggested a conclusion .. that the universe appears to have been designed by a pure mathematician."[50] To this he adds "In one sense it may be argued that everything is mathematical. The simplest form of mathematics is arithmetic, the science of numbers and qualities, and these permeate the whole of life."[51]

In support of this thesis Jeans cites the example of the carbon atom which is "noteworthy chemically as forming a sort of transition between the metals and non-metals". This atom consists of six electrons and differs from "the atoms of boron and nitrogen, only in having one electron more than the former and one electron fewer than the latter. Yet this slight (numerical) difference must account in the last resort for all the difference between life and absence of life."

Pursuing the logic further, he says, "other similar cases are known to Chemistry. Magnetic phenomena appear in a tremendous degree in iron, and in less degree in its neighbours, nickel and Cobalt. The atoms of these elements have 26, 27 and 28 electrons respectively .. Somehow, then, although again mathematical physics has not yet unravelled how, magnetism depends on the peculiar properties of the 26, 27 and 28 electrons in these atoms, especially the first. Radio-activity provides a third instance, being confined, with insignificant exceptions, to atoms having from 83 to 92 electrons; again we do not know why .. Chemistry suggests that, like magnetism and radio-activity, life may be an accidental consequence of the special set of (mathematical) laws by which the present universe is governed."[52]

What is true of the physical universe should be true of the musical cosmos also. If the former is a mathemtical idea assuming tangible forms and qualities under the rule of numbers, so should the latter be and behave.

The Indian concept of the divine origin of music is probably a mythologically expressed recognition of this very truth. A real tribute to this concept can only be to see that this recognition is made to help rather than hinder a mathematical understanding of Indian music.

Such an understanding, as we have noted earlier, can come best through a formulation of certain basic axioms regarding the tonal dynamics and architectonics of melody, expressible in terms of musicometry, the system of measuring tonal forces and charting out tonal paces in mathematical terms. These axioms, better called theorems till they are proved to be correct and based on or derived from meanings or implications of the essential principles of melody upheld or uncontroverted by Indian musicology, should be treated as tentative starting-points rather than as established end-results of the proposed fact-finding mission.

Some of these main theorems may be stated as below,

1. The Graphic symbolisation of the Octave structure of the pitch series is accomplished by equiangular spiral line ascending or decending continually and returning after every turn to points above or below those passed before, the radius of the spiral being exactly doubled or halved as it moves away from or towards the centre of the spiral after having described an angle of 360°, and the equation of the spiral being $r = 2^{\frac{\theta}{\pi}}$

Note:

(a) The equiangular spiral line has been deemed satisfactory as a representation of the acoustical phenomenon, but not as that of the musical phenomenon by Victor Zuckerkandl, who says, "it fails to express the sense of arriving at a destination that we experience when the movement along the scale reaches the octave. Something clicks; but no sign of any clicking shows in the even course of the spiral."[53] But Zuckerkandl seems to have overlooked the fact that the octave and the tonic, and, for that matter, any note and its octave are as a matter of rule found to be in a straight line with the centre of the spiral, which, may geometrically speaking, be called an acoustic representation of the musical clicking.

(b) The Sanskrit equivalent of spiral is *Kundalini*, a well-known term of the Indian Yogic and Tantric systems which often occurs in the *delineation of the methodoloy of self-realization*. Geometrically viewed, the musico-metrical *Kundalini* is a linear movement having a fixed centre, towards and away from which it can extend in a certain defined order from infinity to infinity, but which it can never exactly touch. If this linear movement is visualized as a representation of mental movement and this centre is viewed as a symbol of *AUM*, the centre of all cosmic becoming according to Indian philosophy, it lends countenance to the ancient Indian assumption that music bridges, as nothing else does, the worlds of material perceptions and mystic experiences.

2. The octave structure of the pitch series from any given pitch to its next unison, that is, from any given frequency to its double or half, if divided

in its circular form into seven equal sectors, shall indicate seven tonal zones or *swara grama*-s. In this zonal division the tonic shall be permanently located in the exact middle of the first zone and each one of the remaining six notes of any diatonic mode shall be located somewhere in each of the remaining six zones subject to laws of consonance and dissonance, with the provision that in certain eventualities brought about by laws of symmetry and asymmetry, any two neighbouring notes may be located in the same zone, any of its adjacent zones being left entirely vacant, provided that the middle of the Octave, expressible mathematically as 2, shall not be crossed one way or another whether by the fourth or by the fifth note and, further, that if it has to be crossed at all, the distance traversed either way shall remain below 81/80.

Note:

(a) The fact that the octave had been divided by Indian musicology into seven tonal zones, called after the seven notes, is borne out by Bharat's description of the notes of the *Shadja* and the *Madhyama Grama*-s. The further fact of each of the two *grama*-s having unequal tonal zones, according to Bharata, because of their notes having deviated from their locations of rest due to tonal tension, does not in itself contradict the hypothesis of of the natural equality of the zones when their respective notes are in a state of rest.

(b) The movement of the radius of a spiral has two implications, angular and linear. The former denotes the zonal position of a note while the latter indicates its tonal location in the scale, with reference to the tonic note. In the ancient Indian system zonal measurements were made in terms of irrational numbers and tonal measurements in terms of rational numbers. The system has been reversed in the modern Western system, in which tonality of a note is expressed by irrational numbers and its zonality by rational numbers. The difference between the two systems of measurements being, in ultimate analysis, the difference between melody and harmony, needs careful attention in any comparative study of the musical values of the east and the west.

(c) A *swara* means in Sanskrit something that 'shines with its own light.' Musically speaking a note becomes a *swara* when it happens to possess an element of musical energy given to it by its tonal relation to the tonic note and when that energy has been converted into musical light by virtue of its zonal position in the octave. This dual character of a *swara*, having both linear and angular implications and requiring for its expression both rational and irrational numbers, is a psycho-acoustical phenomenon which needs analysis in the light of the alpha-phonoid/phoneme theory for its proper understanding. Such an analysis may explain why a *swara*, which is purely a temporal entity existing only in time, presents a spatial illusion of being 'high' (*udatta*) 'low' (*anudatta*) and even creates a tactile sensation of being 'flat' (*komala*) or 'sharp' (*teevra*). It may also explain how various *swara*-s, having different tonal and zonal locations in the mathematical sense get blended into one melodic whole in the psychological context, and shed away their identities while retaining their individualities in the melodic flow in which they tend "to be" and "not be" from start to finish.

(d) The hypothesis that the octave should be divided into seven equal zones and that the tonic should be located in the exact middle of the first zone, may not have been explicitly mentioned by Indian musicology, but it is implied in the use of the term *Madhyama,* which means in Sanskrit 'the middle', and which presumably indicates the middle point of the octave, i.e. $\sqrt{2}$, F sharp, of the modern tempered scale. In later Indian musicology the fourth note having the mathematical value of 4/3 has come exclusively to be indentified with the term *Madhyama,* which was called *anashi* by Bharata[55] and *alopi* by Dattila,[56] on ancient Vedic authority. That the real implication of the Vedic authority had been lost by the time of Bharata is not unlikely, because a theory of the unfailing use of the *Madhyama* having the mathematical value of 4/3 carries no sense. So, it may be assumed that *Madhyama* in this context meant the middle point of the octave, that is, the point of dividing the fourth and the fifth zones and was considered to be uncrossable. This indicates that the tonic note was placed at a distance of 180° from it, that is, in the exact middle of the first zone.

(e) Bharata has referred to the incidence of overlappings in notes (*swara*-s) and also in modes (*jati*-s), defining "overlapping" as 'the thing which exists between the two (similar things, partly merging into each other), e.g. the transition of a season."[57] This is suggestive of cases in which a note belonging to one zone comes to be located in another preceding or succeeding zone, bringing about an overlapping of notes and modes. Such overlappings are quite frequent in the second and the third, *Rishabha* and *Gandhara,* and also in the sixth and the seventh, *Dhaivata* and *Nishada* zones. Such overlappings have resulted in same notes being called both *Rishabha* and *Gandhara* or both *Dhaivata* and *Nishada* in the modes of the southern system of Indian music, where the number of the generic modes is as large as seventy-two. In the northern system overlapping has been a taboo, resulting in the reduction of the number of the generic modes to practically ten, as shown by Bhatkhande. Seen from this point-of-view the southern school affords, on the whole, a wider range of melodic possibilities then its northern counterpart, and may, hence, be said to be more catholic in its melodic taste than the latter. The notion that the northern system also provides sometimes, if rarely, for the location of two notes in the same tonal zone is based on a misconception of facts. Two notes appear in one zone, if and when they do so, not in a straight but in a tortuous manner, not boldly but almost apolgetically. The *Madhyama*-s do appear, in *raga*-s like *Lalita,* but of these the second and the higher one is not F sharp (*Teevra Madhyama*) but G flat (*Mridu Pancham*) which fact is often overlooked.

(d) In certain modes the fourth note goes beyond, or the fifth note remains below, the middle of the octave, i.e. $\sqrt{2}$. But in no case is such deviation permitted to exceed 2048/2025, which is less than 81/80, the Pythagorean *Comma diesis* or the *Pramana sruti,* mainly because it was the highest permitted limit of deviation for the crossing of the *Madhyama* by the fourth or the fifth note one way or the other.

3. The octave structure of the pitch series, if divided into twenty-two equal parts, shall indicate twenty-two microtonal zones or *sruti grama*-s. In this zonal division the tonic shall be located at the meeting point of first

and the twenty-second zones and no two adjacent notes of any melody shall be located at a mutual distance of less than one and more than six such *sruti-grama*-s, as demanded by the laws of symmetry and asymmetry.

Note:

(a) Whether the twenty-two microtones (*sruti*-s) of Indian music were of equal or unequal length has since long been a controversial issue. This controversy seems to be a result of our associating the term *sruti* exclusive with the tonal location of a melody and dissociating it altogether from the tonal space of the octave. It may reasonably be that *sruti* was a term applied to both, indicating as much a unit of tonal space as a point of tonal location. When Bharata said that two *swara*-s are mutually consonant when placed at a distance of nine or thirteen *sruti*-s and dissonant when placed at that of two or twenty *sruti*-s[58] he was evidently using the term *sruti* as a fixed unit of tonal space. On the other hand, when he suggested the process of *chatus-sarana* for 'explaining the system of *sruti*',[59] he was using it to denote various separate points of tonal location.

(b) Supposing that the entire octave, from any note to its unison, consists of 360 *degrees*, or 301 *savart*-s or 1,200 cents, or 252 *Manorama*-s (the last being equal to .7 *degrees*, or .21 cents, and approximately equal to 1.2 *savarts*), the distances of thirteen and nine *sruti*-s produce a ratio approximately equal to 2:3 and 3:4, and the distances of twenty and two *sruti*-s produce a ratio approximately of 135:256 and 128:135 respectively between the former and the latter notes. The science of musicometry having been presumably lost by the time of Bharata, he could not be expected to have given a more precise measure of consonance or dissonance in mathematical terms.

(c) The present system of Indian melody still accepts in practice the minimum of one and the maximum of six *sruti*-s as a measure of tonal distance of any two adjacent notes of a diatonic mode. The difference of five *sruti* units in the minimum and the maximum of tonal distance has been considered necessary in the Indian system of melody in order to provide for wide enough latitude of outer asymmetricality in tonal locations in order to intensify the contract of the inner symmetricality of the tonal design as a whole.

(d) In the present northern system of Indian melody the minimum of tonal distance has been raised to two and the maximum lowered to five *sruti*-units. In other words, the lowest ratio betwen any two adjacent notes must not be less than 15:16 and the highest more than 64:75, the permitted latitude of difference between the smallest and largest distances being no more than three *sruti*-units. Musicologically speaking this is a shrinkage in the sphere of balance between asymmetry and symmetry, which is a sign of musical decadence.

4. The spiral line representing the octave structure of the pitch series consists of infinite points of tonal locations, of which only such points are capable of producing melodic notes as are expressible in terms of integers constituted by prime numbers 2, 3 and 5 or their multiples, the smaller integers being capable of producing more consonant and the bigger ones more dissonant tonal relations.

Note:

(a) Indian musicology does not specifically mention the prime numbers 2, 3 and 5, as being the basic constituents of the integers accepted by it, to formulate its tonal relations. But the emphasis laid by it on the six notes, known commonly as the *Panchama* (3/2), the *Madhyama* (4/3), the *Shuddha Gandhara* (5/4), the *Shuddha Dhaivata* 5/3), the *Komala Gandhara* (6/5) and the *Komala Dhaivata* (8/5) in its melodic elaborations and also in its instrumental tunings, implies its acceptance of these numbers as the basis of all the tonal relations of its modal scales. It is likely that the prime number 7 may also have belonged originally to this group. But from all appearance 7 seems to have been dropped out later.

(b) Consonance and dissonance are relative terms being two sides of the same coin, the tonal tension, and the diminishing order of the one is the developing order of the other. But since the incidence of consonance is finite and that of dissonance infinite, the former has its own natural limitations while the latter has to be limited artistically at some point by a system of melody. The Indian system of melody seems to have limited the admissibility of integers generating tonal relations in its scale at 2187/2048, an integer creating a tonal relation described by Pythagoras as 'the howling of the Devil'.

(c) It is highly significant that the ancient Indian music chose integral fractions like 81/80, 25/24 and 256/243 constituted of prime numbers 2, 3 and 5 as the constituents of its tonal order rather than an irrational number like $\sqrt[12]{2}$, which, known as the semitone, happens to be the single constituent unit of the modern western scale of equal temperament. The former choice leaned towards clarity and precision of tonal expression while the latter leans towards richness and variations of tone-colours. That subjective clarity had been held more essential to the purpose of Indian music than objective richness and that in the matter of communication precision had greater value for it than colourfulness, should be accepted as the fundamental criterion of its appreciation. The future applicability of this criterion, therefore, depends much on the question whether Indian music is going to revert to its old *sruti* scale or to proceed towards the modern scale of equal temperament in times to come.

(d) There are known to be two different methods of forming musical scales, the *divisive* and the *cyclic*. The former, as in the West admits a large number of prime numbers as constituents of its tone-making integers. The latter, as in China and Japan, keeps down the number to only two prime numbers 2 and 3. The Indian scale is both *divisive* and *cyclic*. The inclusion of the prime number 5 in this scale and the exclusion of all others, like 11, 13, 17 and so on, is indicative of this dual fact. By being what it is, the Indian scale provides an ideal field for balancing pairs of opposites, like symmetry and asymmetry, consonance and dissonance, definiteness and infinity.

(e) The points of microtonal locations (*sruti-sthana*-s) have been considered, in theory, to be infinite — (*anantah*) but at one time they were as many as sixty-six and later came down to twenty-two, in practice.[60] The number of these points has always depended on their being clearly perceptible — *pratyavagantavyah*.[61] The criterion of perceptibility is a very flexible one, differing from one set of circumstances to another. At present this number, has come down, for all practical purposes, to twelve, which is the number of notes in the present chromatic scale. Invention of scientific instruments should be able to widen the range of tonal perceptibility. A scale, therefore, of forty-three microtones should be not be altogether impossible to construct, and that of twenty-two microtones should be considered quite a practical possibility. (The mathematical values of the proposed forty-three microtones appear in Appendix - I).

5. A diatonic melodic mode comes into existence only when each one of its seven notes is connected with some other note by relations of consonance, either by the rato of 3:2 or of 5:4 or of 6:5. Among these relations the number of the first ratio (3:2) cannot be more than five and should not be less than four; the number of the other two (5:4 and 6:5) depending on how much of a tonal consonant or dissonant content of the mode is desired.

Notes:

(a) "The whole course of our Western music during the last thousand years .. seems reflected in the story of the mutual relation of consonance and dissonance," observes Victor Zuckerkandl, "First, consonance is the only state of sound believed admissible in music, dissonance is banned, Gradually dissonance is admitted and brought under control. Consonance, however, .. is the rule and dissonance the exception, (But) .. things continue to shift. Dissonance takes up most of the space, it becomes the rule, and consonance the exception .. To-day, dissonance has become the normal state of sound".[62] Dissonance tended to grow in Indian music also, and for artistically justifiable reasons. But the predominance of the tonic (the *Shadja*) on the one hand, and the compulsory presence of either the *Madhyama* (4/3) or the *Panchama* (3/2) on the other, has kept its growth under healthy control so far.

(b) The ancient term *grama* has been elaborately discussed in musicology, but its real implication in the over-all scheme of Indian music still remains hazy. It may not be incorrect to assume in this connection that the diatonic modes of the *Shadja Grama* were those in which the *Panchama* (3/2) was unfailingly present and that those of the *Madhyama Grama* invariable contained the *Shuddha Madhyama* (4/3). This assumption is supported by Bharata's statement that "in the *Madhyama grama*, *Panchama* and *Rishabha* are consonant while *Shadja* and *Panchama* are consonant in the *Shadja grama* (only)".[63] But there are reasons to support that there were also certain modes in which none of the above two notes had any place. In these modes either one of the two *Gandhara*-s (5/4 and 6/5) or one of the two *Dhaivata*-s (5/3 and 8/5) gas compulsorily present. These were, then most probably the modes of the *Gandhara* and the *Dhaivata Grama*-s, which went out of vogue later on account of their high content of dissonance. It may be added in this connection that though the Sanskrit treatises mention only the *Gandhara Grama* (which term probably included the *Dhaivata Grama*), certain ancient Tamil works belonging to the third century A.D. do refer to four *grama*-s.[64] This means that the almost exclusive survival of the modes of the *Shadja Grama* implies the ultimate predominance of consonance over dissonance in relation to the tonic (the *Shadja*), proclaiming its unquestionable supremacy in Indian music.

6. A melodic mode being constituted not by any of its notes individually but by the composite pattern of all its note-intervals in their totality, it is born wherever, that is at the beginning of whichsoever note-interval, its tonic may happen to be located, with the added fact that each one of such locations, the number of locations depending on the number of the note of the pattern, produces a different mode with a different ethos, and what is true of a pattern is also true of the reverse of it.

Notes:

(a) Dattila, who is supposed to be one of the sons of Bharata but who probably flourished before him, says that the tonic of a mode may be located at any of the microtonal points of the octave spiral without in any way affecting the over-all compositions of the mode.[65] Since the central term of reference for all the notes of a mode is its tonic, the latter unfailingly creates the mode inherent in the pattern, wheresoever it may be located in the octave spiral. This means that the location of the tonic is not an absolute but a relative concept in Indian music and that the ensuring modes are algebraical rather than arthimetical phenomena. (See Appendix - III).

(b) The fact that the shifting of the tonic from one note-interval to another produces an altogether different mode is one of the central fats of the melodic system, called in Indian musicology as *moorcchana,* which means the brightening up by turns of each one of the many aspects of the melodic content contained in any given note-pattern. Since multiplicity of aspects in a note-pattern is possible mathematically only when the note-intervals are of unequal lengths, the chromatic scale of all the twelve notes, which has an unchanging character with uniform note-intervals, is totally unsuited for Indian music. Only the presence of asymmetry in symmetry can bring out individuality in a community. Indian music believes in creating only such tonal communities as are composed of individualities distinguishable from one another and it permits the community to speak only through one individual at a time.

(c) The reverse of a note-pattern is also a note-pattern, because the same laws of consonance-dissonance and symmetry-asymmetry relations enter into the composition of both. This gives birth to tonal individualities in a tonal community, which are not only different from but also the reverses of one another. This balance between freedom and law of tonal co-existence is the soul of Indian music, which must always express itself in the elaboration of all its norms and their deviants.

7. The tonic has one fixed character but the other six notes have many changing ones. Each mode should, therefore, be formulated and known in terms of the characters of all its notes taken as a whole.

Notes:

(a) The simultaneous division of the octave in seven *swara*-zones and twenty-two *sruti*-zones creates the varying characters of the six tones, which are described in spatial terms as being 'high' or 'low' and also in tactile terms as being flat (*mridu*) or sharp (*teevra*). Tones being mere abstractions their characters are nothing but illusory, still the illusion is undeniably real and a real matter of unmistakable perception while it lasts.

(b) The nothern and the southern systems of Indian music take different views of the tonal characters and musicologists have differed from one another in defining them. But the differences are merely superficial and do not in any way interfere with the common acceptance by both the systems of the seven *swara*-s and the twenty-two *sruti*-s. It is not a little unfortunate that in later ages the differences have come to be emphasized more than the common points and that the two systems have been less attentive to their common laws than to their divergent conventions.

(c) A much talked of difference between the two systems lies in their acceptance of the standard scale, which is *Bilaval* in the north and *Kanakangi* in the south. Whatever be the history of this difference, which has been presumably an outcome of the differences of tonal nomenclatures, the fact remains that the standard scale, if one is at all needed, should be one of the two scales of *Shadja* and *Madhyama grama*-s, mentioned by Bharata, known as the scale of *Kafi* in the north and of *Kharaharapriya* in the south. This scale represents the minimum tonal deviations of the six tones in their states of tonal tension from their states of rest. Though Bharata has not mentioned the reason why this was accepted by him as the standard scale, the reason can be easily discovered by its musicometrical analysis.

The above theorems may be followed by several others for a more precise and coherent understanding of the Indian system of melody. Though, as Mendelssohn once said, "the tonal language is too precise to be translated into words," and music, according to Walter Pater, 'is the least tangible of all arts', still the fact stands, as St. Ausgustine remarked long ago, that "music is ordered motion'. The disciplined dynamism of music, generates, according to the 'motor theory' of melody, a complex of emotions through tonal motions,[66] and "goes to the roots of our being and takes shapes in the inner gestures which embody our deepest and most intimate responses"[67]. In the words of John Dewey, "Music .. illustrates perhaps better than any other art the sense in which form is the moving integration of experience"[68].

The knowledge of the correlation between musical form and emotional experience has yet to pass through lots of objective experimentations in order to attain the dignity of a science. But the emergence of this science, if and when it takes places, shall depend very much on an exact knowledge of musical forms, which can be acquired best through musicometry.

"Ever since chaos was conquered", says Percy Goetschus, "there has been no phenomenon in nature without form .. Music will not be exempted from its obligation to comply with the law.[69] To this one may add that musicologists will not be exempted from their obligation to explore the 'law' in its minutest details through means that are both subjectively and objectively legitimate.

APPENDIX — I

Main Notes in the Scale of Indian Music in 22 *Sruti* Zones and 7 *Swara* Zones measured in Savarats, Cents, Degrees and *Manorama* s.

Serial No.	*Sruti* Zones	*Swara* Zones	Intervals	*Sruti* Values	Savarts	Cents	Degrees	*Manorama*-s
1	1	Shadja (*Sa*)	The Tonic or The Fundamental	1/1	0	0	0	0
2	2	Rishbha I(*Re*)	Limma	526/243	22.66	90	27.1	18.97
3	"	" "	Large Limma	135/128	23.083	92.35	27.61	19.327
4	3	Rishabha II (*RE*)	Major Halftone	16/15	28.056	111.65	33.55	23.485
5	"	" "	Large Halftone	27/25	33.449	113.3	40	28
6	4	Rishabha III (Re)	Minor Tone	10/9	45.743	182.35	54.71	38.297
7	4	Rishabha III (Re)	Major Tone	9/8	51.136	204	61.16	42.812
8	5	Rishabha IV (Re)ı	Large Tone	256/223	56.529	223.3	67.11	46.97
9	6	Gandhara I (*GA*)	Small Minor Third	75/64	68.826	274.7	82.32	57.624
10	"	" "	Thihemiton	32/27	73.796	294	88.26	61.782
11	"	" "	Large Thihemiton	1215/1024	74.219	296.35	88.77	62.139
12	"	" "	Minor Third	6/5	79.192	315.65	94.71	66.297
13	7	Gandhara III (Ga)	Third	1000/81	91.486	364.7	109.42	76.594
14	8	Gandhara III(Gaı)	Major Third	5/4	96.879	386.35	115.87	81.109
15	"	" "	Small Ditone	512/405	101.652	405.65	121.81	85.267
16	"	" "	Ditone	81/64	102.272	408	122.32	85.624
17	8	" "	Large Major Third	32/25	107.245	427.3	128.26	89.782
18	9	Madhyama I (*Ma*)	Small Fourth	657/512	119.962	478.7	143.48	100.436
19	10	Madhyama II (Ma)	Fourth	4/3	124.935	498	149.42	104.594
20	"	" "	Large Fourth	27/20	130.328	519.65	155.87	109.109
21	11	Madhyama III (Ma^1)	Augmented Fourth	25/18	142.625	568.7	170.58	119.406
22	"	" "	Harmonic Tritone	45/32	148.018	590.35	177.03	123.921

Serial No.	*Sruti* Zones	*Swara* Zones	Notes	*Sruti* Values	Savarts	Cents	Degrees	*Manoram-s*
23	12	Panchama I (*Pa*)	Cyclic Tritone	64/45	152.982	609.65	182.97	128.079
24	,,	,, ,,	Small DimiLished Fifth	36/25	158.375	641.3	189.42	132.594
25	13	Panchama II (Pa)	Diminished Fifth	40/27	170.672	680.35	204.13	142.891
26	13	Panchama II (Pa)	Fifth	3/2	176.065	702	210.58	147.406
27	14	Panchama II (Pa^{I})	Large Fifth	1024/675	181.038	721.3	216.52	151.564
28	15	Dhaivata I (*Dha*)	Small Diminished Sixth	25/16	193.755	772.7	231.74	162.218
29	,,	,, ,,	Diminished Sixth	128/81	198.728	792	237.68	164.376
30	,,	,, ,,	,, ,,	405/256	199.148	794.35	238.19	166.733
31	,,	,, ,,	Small Sixth	8/5	204.121	813.65	244.13	170.891
32	16	Dhaivata II (*Dha*)	Sixth	81t50	209.514	835.3	250.58	175.406
33	17	Dhaivata III (Dha^{I})	Harmonic Sixth	5/3	221.808	884.35	265.29	185.703
34	,,	,, ,,	Augmented Sixth	2048/1215	226.781	903.65	271.23	189.861
35	,,	,, ,,	Cyclic Sixth	17/16	227.204	906	271.74	190.218
36	17	,, ,,	Large Sixth	128/75	232.174	925.3	277.68	194.376
37	18	Nishada I (*Ni*)	Seventh Harmonic	225/128	244.471	976.7	292.9	205.03
38	19	Nishada II (*Ni*)	Small Minor Seventh	16/9	249.864	926	298.84	209.188
39	,,	,, ,,	Minor eventh	9/5	255.257	1017.65	305.29	213.703
40	20	Nishada III (Ni)	Seventh	50/27	267.551	1066.7	320	224
41	20	Nishada III (Ni)	Major Seventh	15/8	272.944	1088.35	326.45	228.515
42	21	Nishada IV (Ni^{I})	ugmented Major Seventh	256/135	277.917	1107.65	332.39	232.673
43	,,	,, ,,	Cyclic Major Seventh	243/128	278.34	1110	332.9	233.03
44	22	Tara Shadja (*Sa*)	Octave	2	301	1200	360	252

Note: The present chart shows the Octave divided into Savarts, Cents, Degrees and *Manorama*-s.
The Octave = 301 Savarts, or 1200 Cents, or 360 Degrees, or 252 *Manorama*-s.

APPENDIX — II

MODAL CYCLES (*Moorchhana* — *Chakra*-s

	Tempered Scale	*Sruti Scale*		*Intervals*
1.	1 - 3 - 1 - 2 - 1 - 3 - 1	2 - 5 - 2 - 4 - 2 - 5 - 2	(a)	16/15 x 75/64 x 16/15 x 9/8 x 16/15 x 75/64 x 75/64
			(b)	135/128 x 32/27 x 16/15 x 9/8 x 135/128 x 32/27 x 16/15
			(c)	16/15 x 32/27 x 135/128 x 9/8 x 16/15 x 32/27 x 135/12
2.	1 - 3 - 1 - 1 - 2 - 3 - 1	2 - 5 - 2 - 2 - 4 - 5 - 2	(a)	16/15 x 75/64 x 16/15 x 16/15 x 9/8 x 75/64 x 16/158
			(b)	16/15 x 32/27 x 135/128 x 16/15 x 9/8 x 32/27 x 135/128
3.	1 - 3 - 2 - 1 - 1 - 3 - 1	2 - 5 - 4 - 2 - 2 - 5 - 2	(a)	16/15 x 75/64 x 9/8 x 16/15 x 16/15 x 75/64 x 16/15
			(b)	135/128 x 32/27 x 9/8 x 16/15 x 135/128 x 32/27 x 16/15
4.	1 - 3 - 1 - 2 - 2 - 2 - 1	2 - 5 - 2 - 4 - 3 - 4 - 2	(a)	16/15 x 75/64 x 16/15 x 9/8 x 10/9 x 9/8 x 16/15
			(b)	135/128 x 32/27 x 16/15 x 9/8 x 10/9 x 9/8 x 16/15
5.	1 - 2 - 2 - 2 - 1 - 3 - 1	2 - 4 - 3 - 4 - 2 - 5 - 2	(a)	16/15 x 9/8 x 10/9 x 9/8 x 16/15 x 75/64 x 16/15
			(b)	16/15 x 9/8 x 10/9 x 16/15 x 16/15 x 32/27 x 135/128
7.	1 - 3 - 3 - 2 - 2 - 1 - 2	2 - 5 - 2 - 4 - 3 - 2 - 4	(a)	16/15 x 75/64 x 16/15 x 9/8 x 10/9 x 9/8 x 16/15
			(b)	135/128 x 32/27 x 16/15 x 9/8 x 10/9 x 9/8 x 16/15
8.	2 - 2 - 1 - 1 - 3 - 2 - 1	3 - 4 - 2 - 2 - 5 - 4 - 2	(a)	10/9 x 9/8 x 16/15 x 135/128 x 32/27 x 9/8 x 16/15
			(b)	10/9 x 9/8 x 16/15 x 16/15 x 75/64 x 9/8 x 16/15
9.	1 - 2 - 3 - 1 - 1 - 2 - 2	2 - 4 - 5 - 2 - 2 - 4 - 3	(a)	16/15 x 9/8 x 32/27 x 135/128 x 16/15 x 9/8 x 10/9
			(b)	16/15 x 9/8 x 75/64 x 16/15 x 16/15 x 9/8 x 10/9
10.	1 - 2 - 1 - 2 - 2 - 2 - 2	2 - 4 - 2 - 3 - 4 - 4 - 3		16/15 x 9t8 x 16/15 x 10/9 x 9/8 x 9/8 x 10/9
11.	2 - 2 - 1 - 2 - 2 - 2 - 1	(a) 4 - 3 - 2 - 4 - 3 - 4 - 2		9/8 x 10/9 x 16/15 x 9/8 x 10/9 x 9/8 x 16/15
		(b) 3 - 4 - 2 - 4 - 3 - 4 - 2		10/9 x 9/8 x 16/15 x 9/8 x 10/9 x 9/8 x 16/15
		(c) 4 - 3 - 2 - 4 - 4 - 3 - 2		9/8 x 10/9 x 16/15 x 9/8 x 9/8 x 10/9 x 16/15
		(d) 4 - 4 - 2 - 3 - 4 - 3 - 2		9/8 x 9/8 x 16/15 x 10/9 x 9/8 x 10/9 x 16/15
		(e) 3 - 4 - 2 - 3 - 4 - 4 - 2		10/9 x 9/8 x 16/15 x 10/9 x 9/8 x 9/8 x 16/15
		(f) 3 - 4 - 2 - 3 - 4 - 3 - 3		10/9 x 9/8 x 16/15 x 10/9 x 9/8 x 10/9 x 27/25
		(g) 4 - 3 - 3 - 3 - 4 - 3 - 2		9/8 x 10/9 x 27/25 x 10/9 x 9/8 x 10/9 x 16/15
		(h) 3 - 3 - 3 - 3 - 4 - 3 - 3		10/9 x 10/9 x 27/25 x 10/9 x 9/8 x 10/9 x 27/25
		(i) 4 - 3 - 3 - 3 - 3 - 4 - 2		9/8 x 10/9 x 27/25 x 10/9 x 10/9 x 9/8 x 16/15
		(j) 3 - 4 - 2 - 4 - 3 - 3 - 3		10/9 x 9/8 x 16/15 x 9/8 x 10/9 x 10/9 x 27/25

NOTES ON ILLUSTRATIONS

(1) **The Equiangular Spiral** is a spiral in which the radius is doubled or halved as it moves away from the centre or towards it after describing an angle of 360°. Representing, as the spiral does, the pitch-series of the octave, it has been divided into 7 equal Tonal Zones (*swara-grama*-s) and 22 equal Microtonal Zones (*sruti-grama*-s), wherein Microtonal Locations (*sruti-sthana*-s) are shown in terms of integral fractions indicating their ratios with the tonic (1). Thick lines connecting these locations with the centre of the spiral indicate major microtones, while thin lines indicate minor microtones.

(2) **The Seven Tonal Zones** of the octave, called after the seven notes, are primarily all equal in dimension. But their dimensions vary according to variations in the tonal structures of modal cycles (*moorchhana-chakra*-s), the number of which is thirty according to our present calculations. These variations produce tonal overlappings on account of which any given note, except the tonic note, may be said to belong to two different zones. This has been called *swara-sadharana* by Bharata.
The octave consisting of 360°, or Ms (*Manorama*-s), each tonal Zones in its natural state consists of 51.4° or 36 Ms. But under the impact of certain modal cycles the variations in zonal dimensions may occur as below:—

(a) *Rishabha* — 135/128 to 32/27 (63.56° or 42.455 Ms)
(b) *Gandhara* — 9/8 to 4/3 (88.26° or 61.782 Ms)
(c) *Madhyama* — 81/64 to 64/45 (63.56° or 42.455 Ms)
(d) *Panchama* — 45/32 to 128/81 (63.56° or 42.455 Ms)
(c) *Dhaivata* — 3/2 to 16/9 (88.26° or 61.782 Ms)
(d) *Nishada* — 27/16 to 256/135 (63.56° or 42.455 Ms)

It may be noted that the range of tonal overlapping between *Madhyama* and *Panchama* is smallest being 2048/2025 (5.94° or 4.148 Ms). Possibly even this small overlapping between *Madhyama* and *Panchama* was not permitted originally, in view of the fact that *Madhyama*, the middle point of the octave, was considered to be indestructible (*alopee* or *avinasi*), and hence uncrossable. Later on this overlapping may have been permitted to the extent of 2048/2025, which is smaller than 81/80, the *pramana sruti* of Bharata.

(3) **The Modal Cycles** (*Moorchhana - Chakra*-s) of Bharat's *Shadja* and *Madhyama Grama*-s are plotted on the Equiangular Spiral of the pitch Series of the Octave in terms of consonant tonal relations in illustrations, 3A, 3B, 4A and 4B. In the former 2 ills. there are *shadja-panchama bhava*-s (tonic-fifth relations), and in the latter 2 ills. there are *shadja-gandhara bhava*-s (tonic-third relations). The former denotes the ratio of 3:2 or 4:3, and the latter denotes that of 5:4 of 8:5; or 6:5 or 5:3. The point to be noted is the difference caused in the order and quantum of these relations by the minute difference in one of theseven notes of one *pramana sruti* — 81/80.

The modes (*raga*-s) born of each of the two modal cycles are *Bilaval, Kafi, Bhairavi, Kalyan, Khammaj, Asavari* and *Lalit Bhairavi* respectively.

The *shadja-panchama bhava*-s in each cycle are five in number, but their order of occurrence is changed. The *shadja-gandhara bhava*-s are four in the *shadja grama* and six in the *Madhyama Grama*. This means that each of the seven modes will require two different ways of elaboration (*prastara*) according to the two different orders of consonance in the two modal cycles. Another important point to be noted is the difference occuring in the denominators of the two series of modes. In the *Shadja Grama* the denominators are 48, 54, 60, 64, 72, 81 and 90 respectively, and in the *Madhyama Grama* they are 24, 27, 30, 32, 36, 40 and 45 respectively.

(4) **The Basic Melodic Arcs** of the generic mode, called *Kafi* in the north and *Kharaharapriya* in the south, are shown in ills. Nos. 5 and 6 in two varieties, the former of the *Shadja Grama* and the latter of the *Madhyama Grama* tonal order as mentioned by Bharata. The denominators of the two tonal orders of *Kafi* or *Kharaharapriya* are 54 and 27 respectively. So, the melodic arcs of the two different orders show the divisions of the two different numbers. The division shows how principles of balancing symmetry and asymmetry operate in India music. The serialisation of melodic arcs produces variations of melodic architecture, which in its turn gives birth to infinite melodic phrases. The basic series of these melodic arcs are as below:—

(a) *SHADJA GRAMA*

Notes : *S — R — G — M — P — D — N — S*
Unit Intervals : 6 + 4 + 8 + 9 + 9 + 6 + 12 = 54

		Intervals	*Notes*
Melodic Arcs :	(1)	27 + 27	*S — sP — S*
	(2)	18 + 18 + 18	*S — M — D — S*
	(3)	18 + 9 + 9 + 18	*S — M — P — D — S*
	(4)	6 + 21 + 9 + 18	*S — R — P — D — S*
	(5)	6 + 12 + 18 + 18	*S — R — M — D — S*
	(6)	6 + 12 + 9 + 27	*S — R — M — P — S*
	(7)	18 + 18 + 6 + 12	*S — M — D — N — S*
	(8)	10 + 8 + 24 + 12	*S — G — M — N — S*
	(9)	10 + 8 + 18 + 18	*S — G — M — D — S*
	(10)	18 + 9 + 15 + 12	*S — M — P — N — S*
	(11)	6 + 21 + 15 + 12	*S — R — P — N — S*
	(12)	6 + 12 + 24 + 12	*S — G — M — P — S*

(b) *MADHYAMA GRAMA*

Notes : *S — R — G — M — P — D — N — S*
Unit Intervals: 3 + 2 + 4 + 4 + 5 + 3 + 6 =27

	Intervals	*Notes*
(1)	9 + 9 + 9	*S — M — D — S*
(2)	9 + 4 + 5 + 9	*S — M — P — D — S*
(3)	3 + 6 + 9 + 9	*S — R — M — D — S*
(4)	3 + 10 + 5 + 9	*S — R — P — D — S*
(5)	9 + 9 + 3 + 6	*S — M — D — N — S*
(6)	5 + 4 + 12 + 6	*S — G — M — N — S*
(7)	5 + 4 + 9 + 9	*S — G — M — D — S*
(8)	9 + 4 + 8 + 6	*S — M — P — N — S*
(9)	3 + 10 + 8 + 6	*S — R — P — N — S*
(10)	3 + 6 + 12 + 6	*S — R — M — N — S*
(11)	5 + 8 + 8 + 6	*S — G — P — N — S*
(12)	5 + 8 + 5 + 9	*S — G — P — D — S*

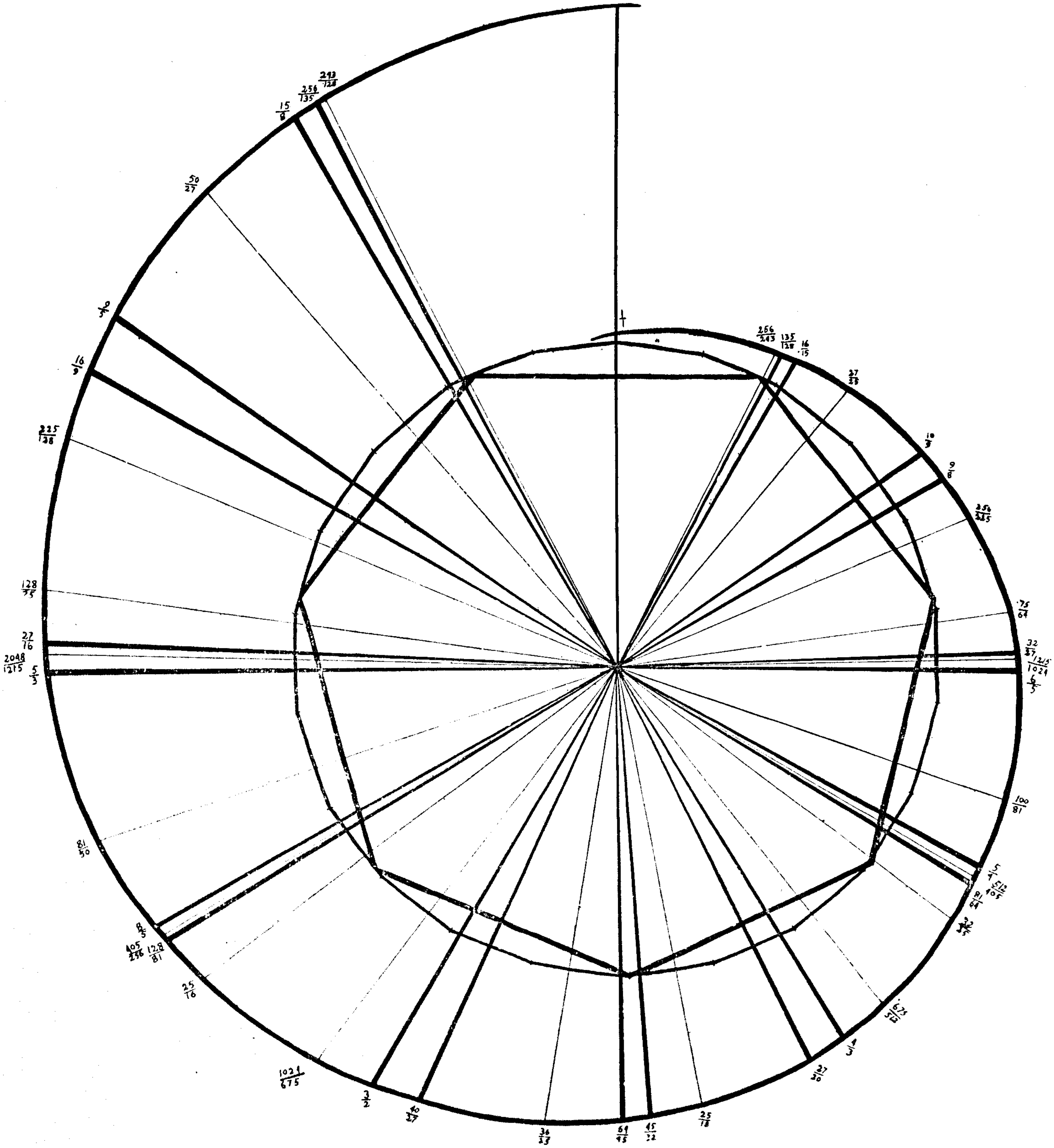


Fig. 1. The eqiuangular spiral of the pitch servies of the Octave.

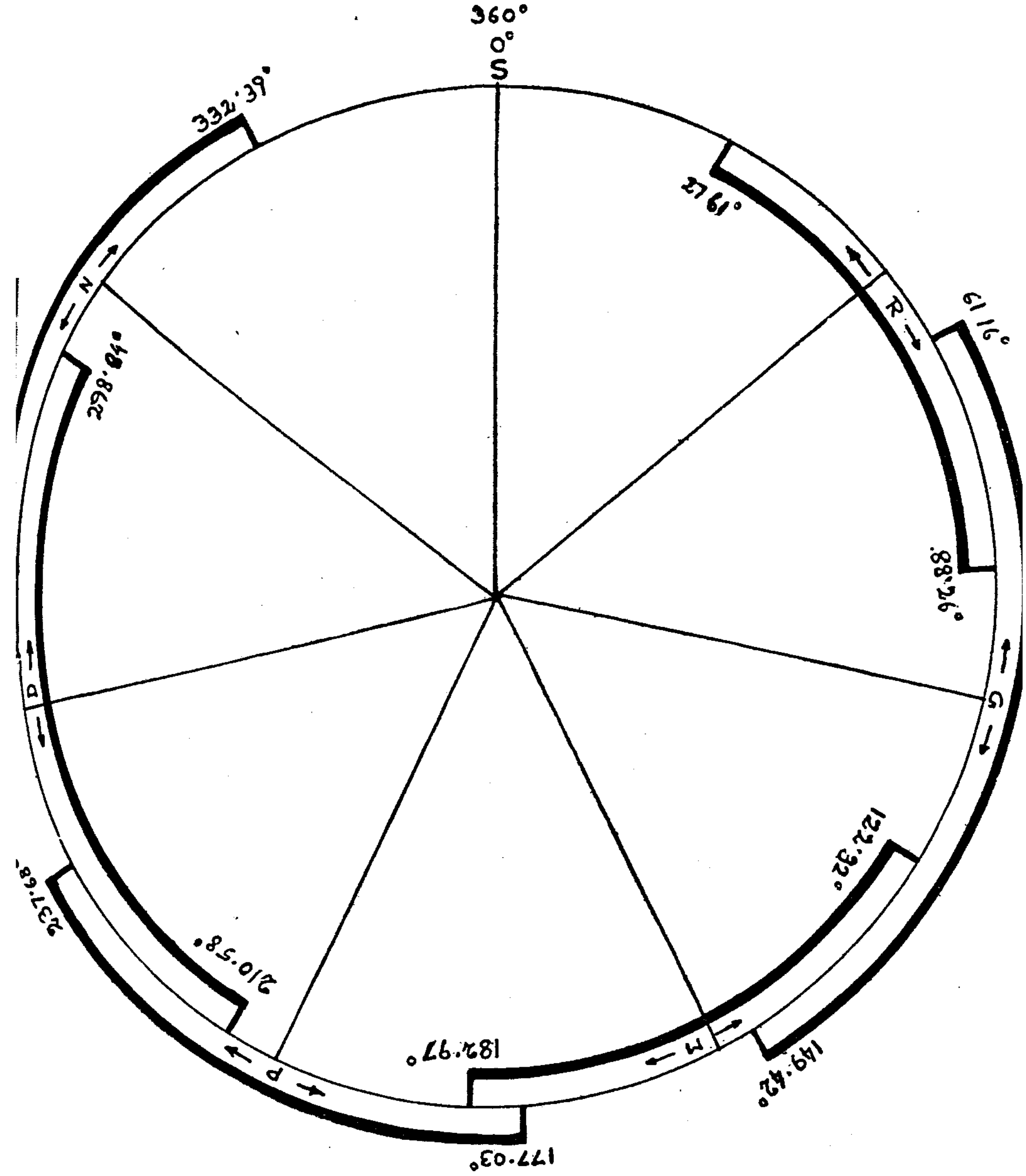
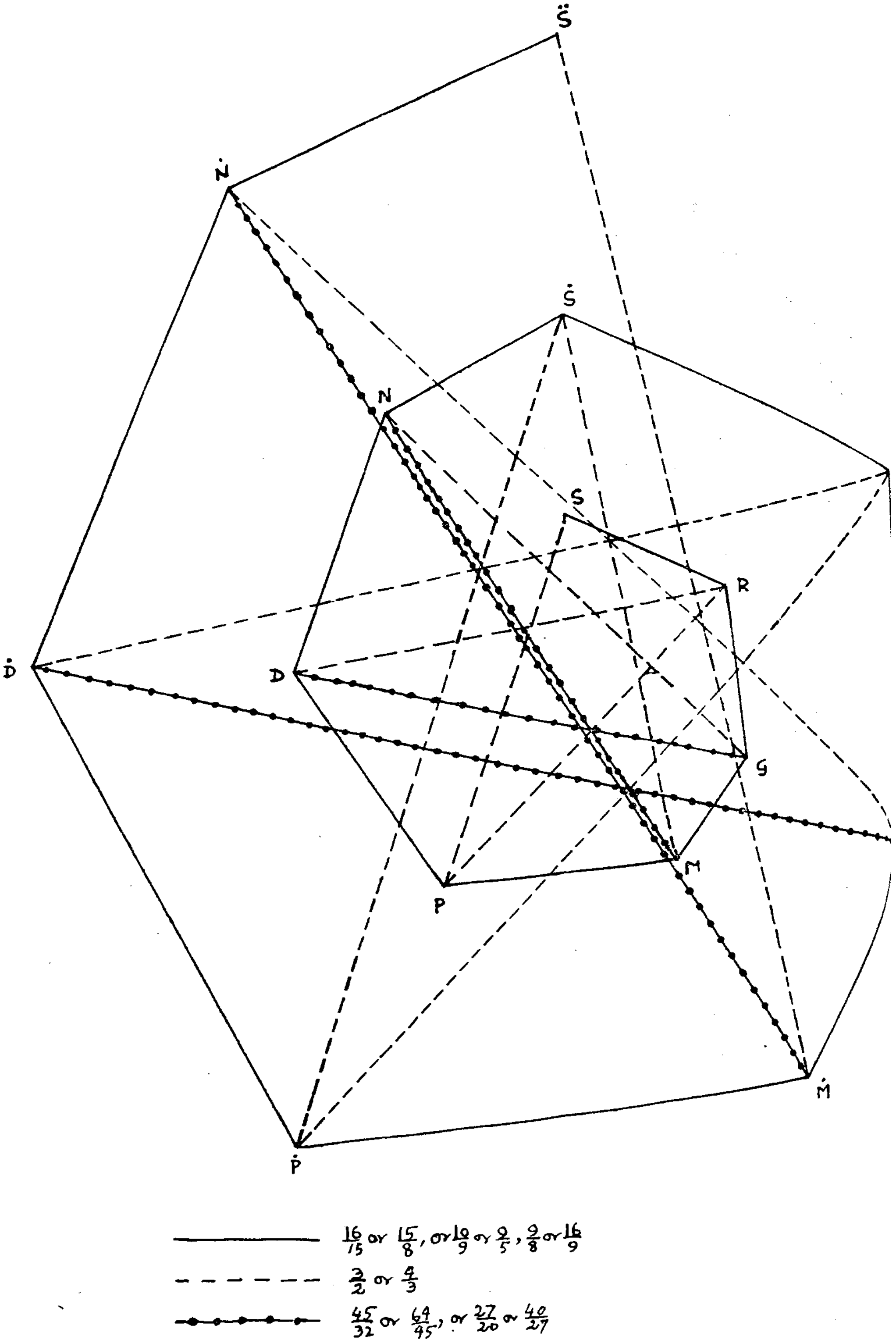


Fig. 2. The seven tonal zones (Svara grama-s)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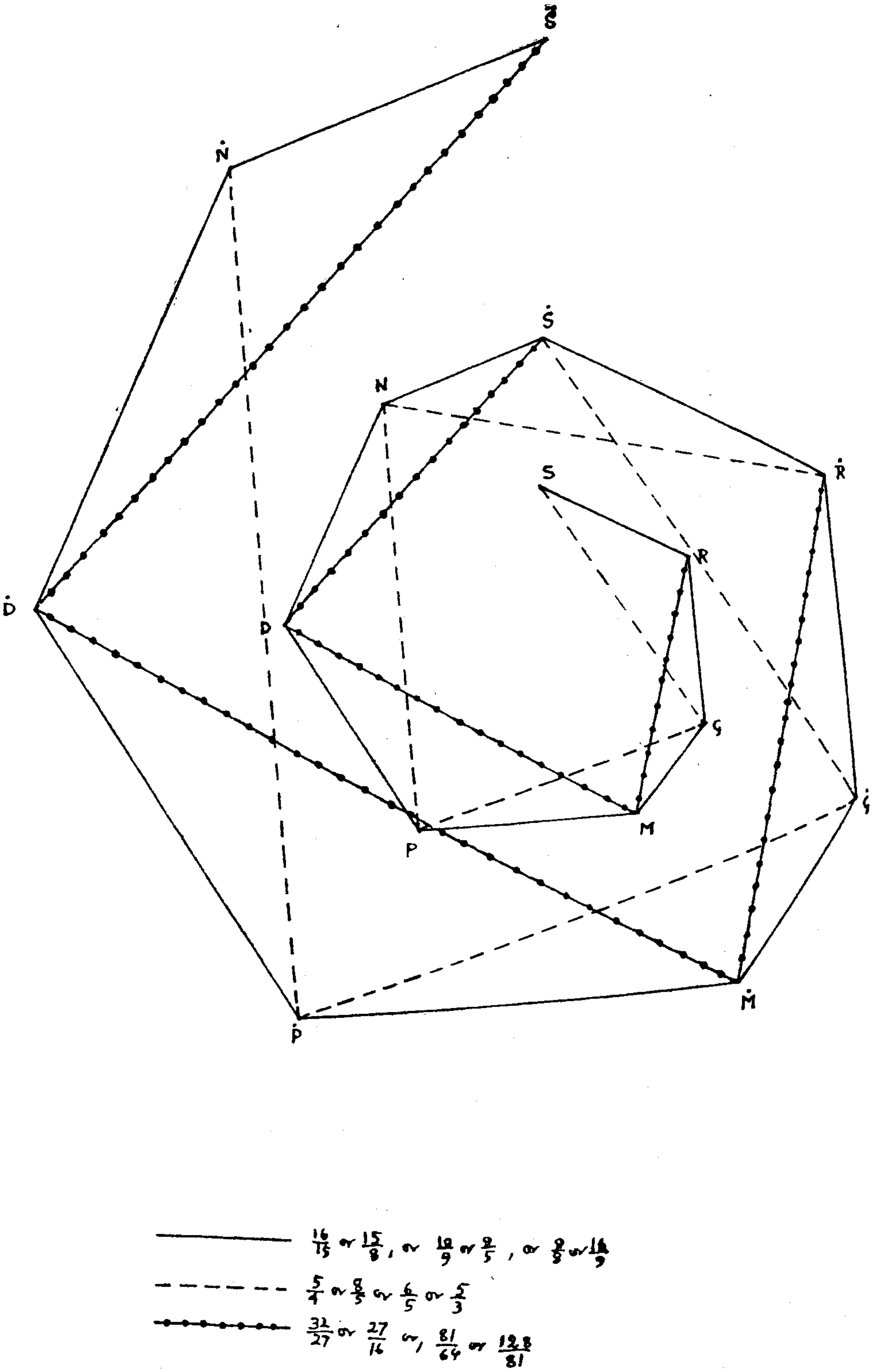
3/2 or 4/3
45/32 or 64/45, or 27/20 or 40/27

Fig. 3a. The modal cycle (moorcchana chakra) of Bharata's shadja grama in shadja-panchama or tonic fifth relations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5/4 or 8/5 or 6/5 or 5/3
32/27 or 27/16 or, 81/64 or 128/81

Fig. 4 a. The modal cycle (moorcchana chakra) of Bharata's madhyama grama in shadja-pauchama or tonic-fifth relations.

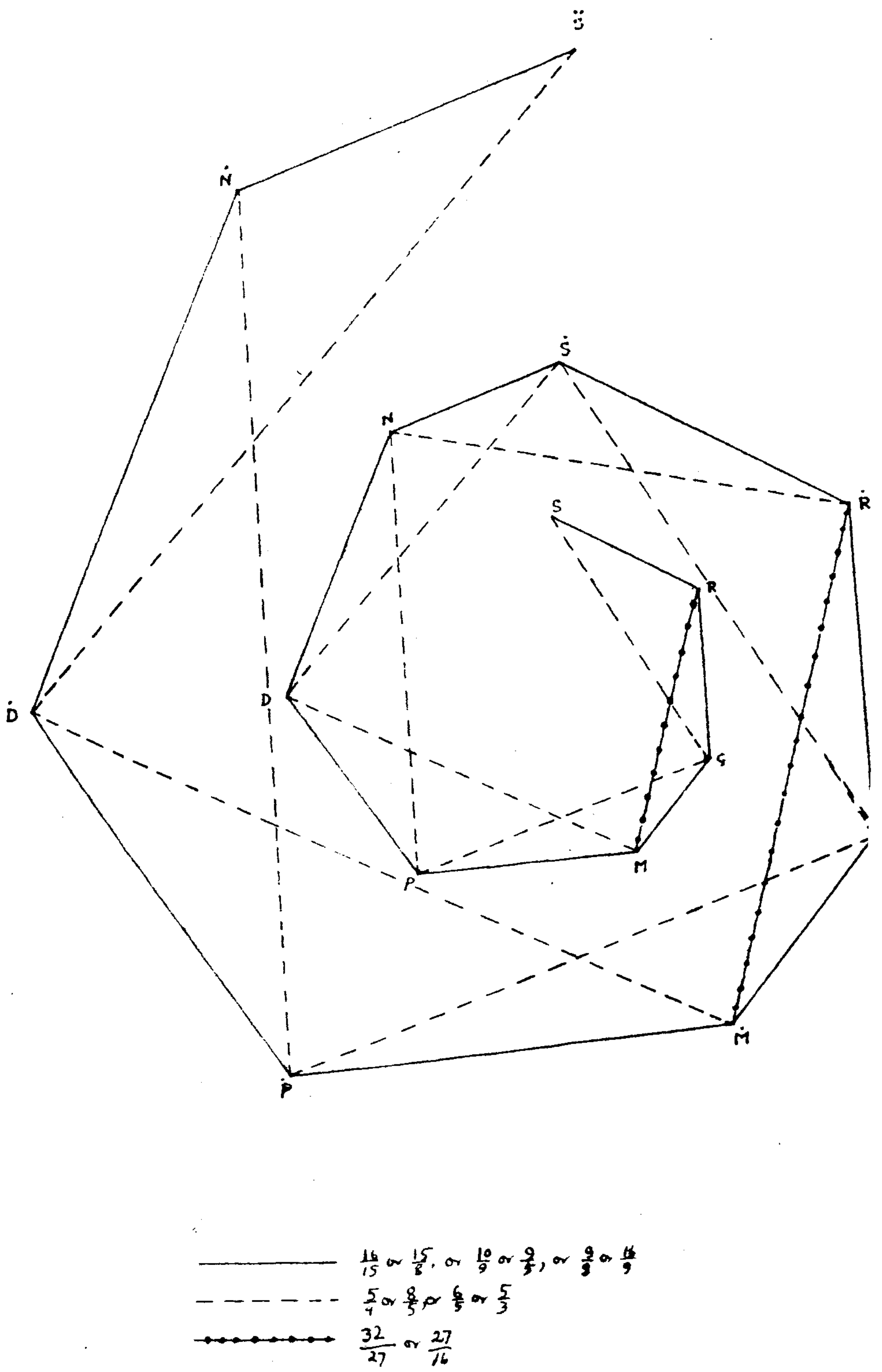
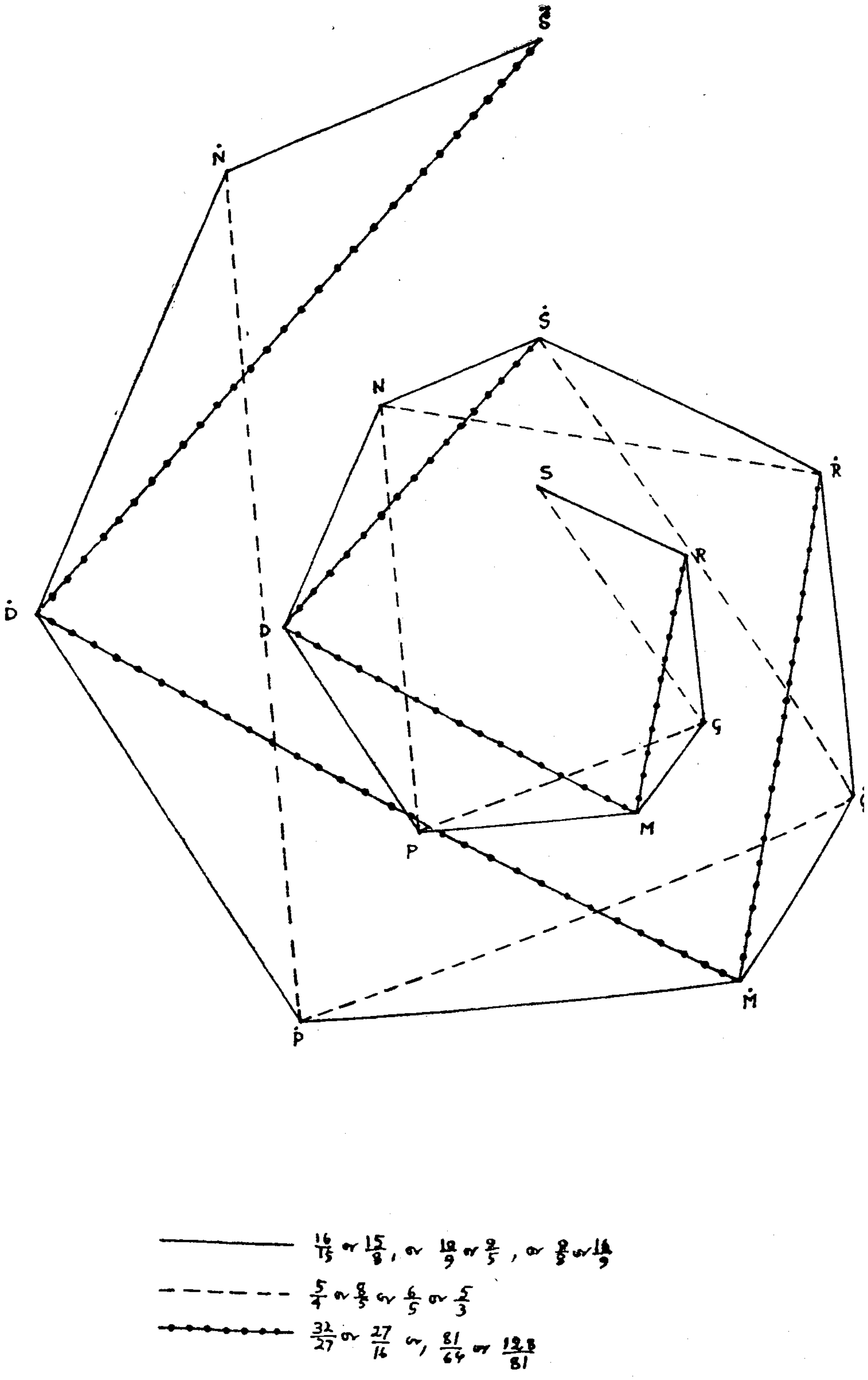


Fig. 4 b. The modal cycle (moorcchana chakra) of Bharata's madhyama grama in shadja-gandhara or tonic—third relations.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5/4 or 8/5 or 6/5 or 5/3
32/27 or 27/16 or, 81/64 or 128/81

Fig. 4 a. The modal cycle (moorcchana chakra) of Bharata's madhyama grama in shadja-pauchama or tonic-fifth relations.

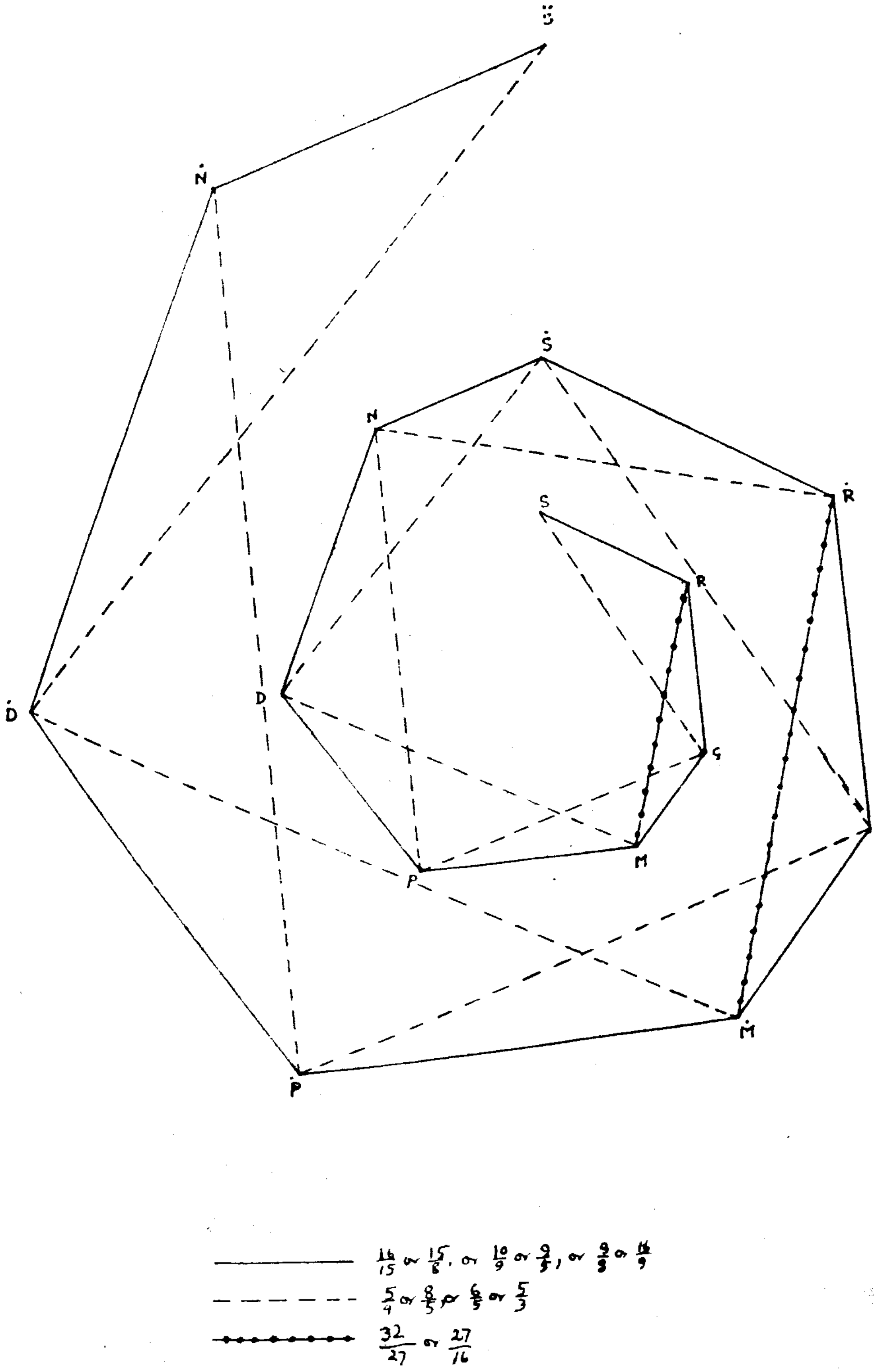

Fig. 4 b. The modal cycle (moorcchana chakra) of Bharata's madhyama grama in shadja-gandhara or tonic—third relations.

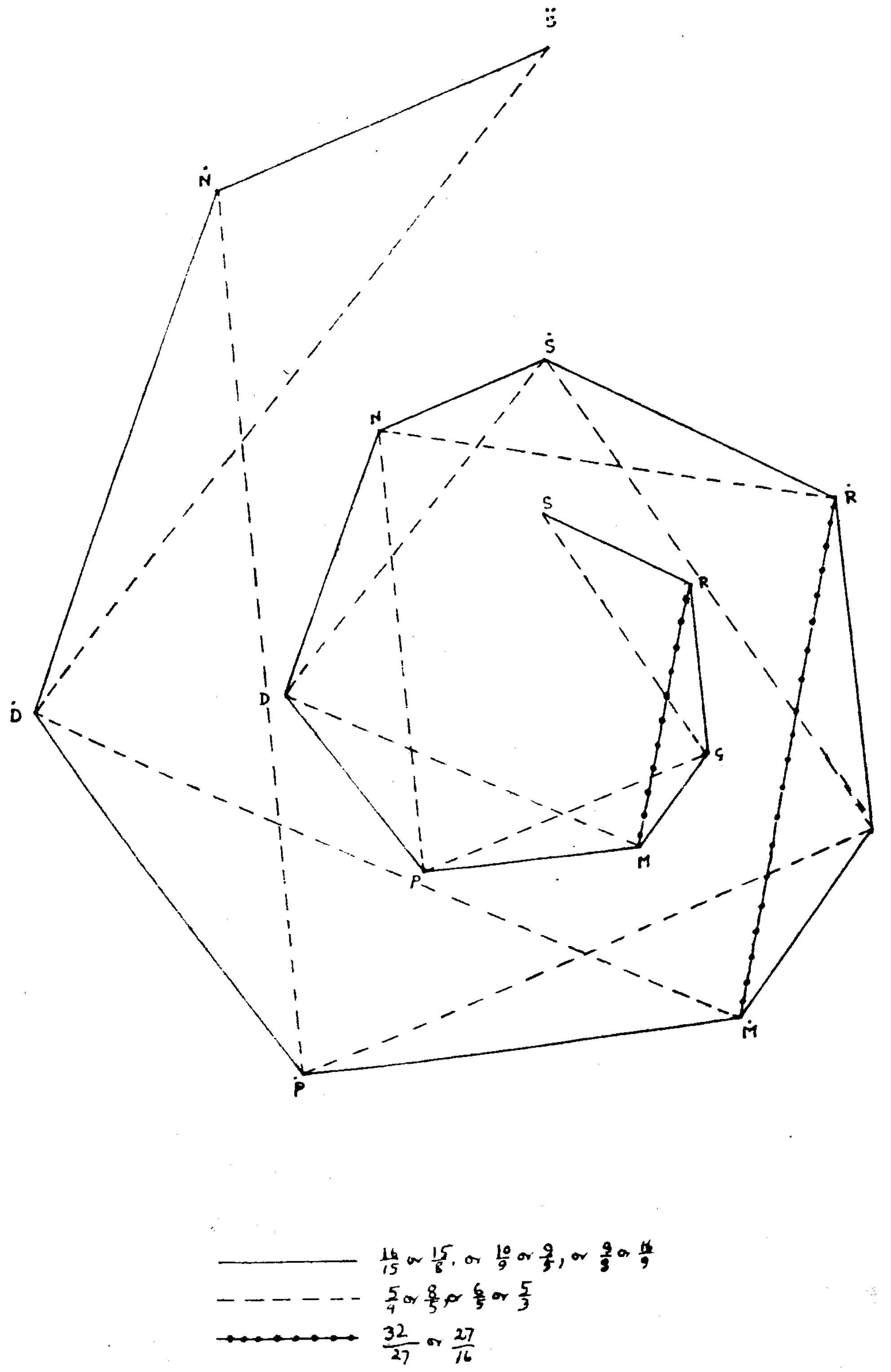

Fig. 4 b. The modal cycle (moorcchana chakra) of Bharata's madhyama grama in shadja-gandhara or tonic—third relations.

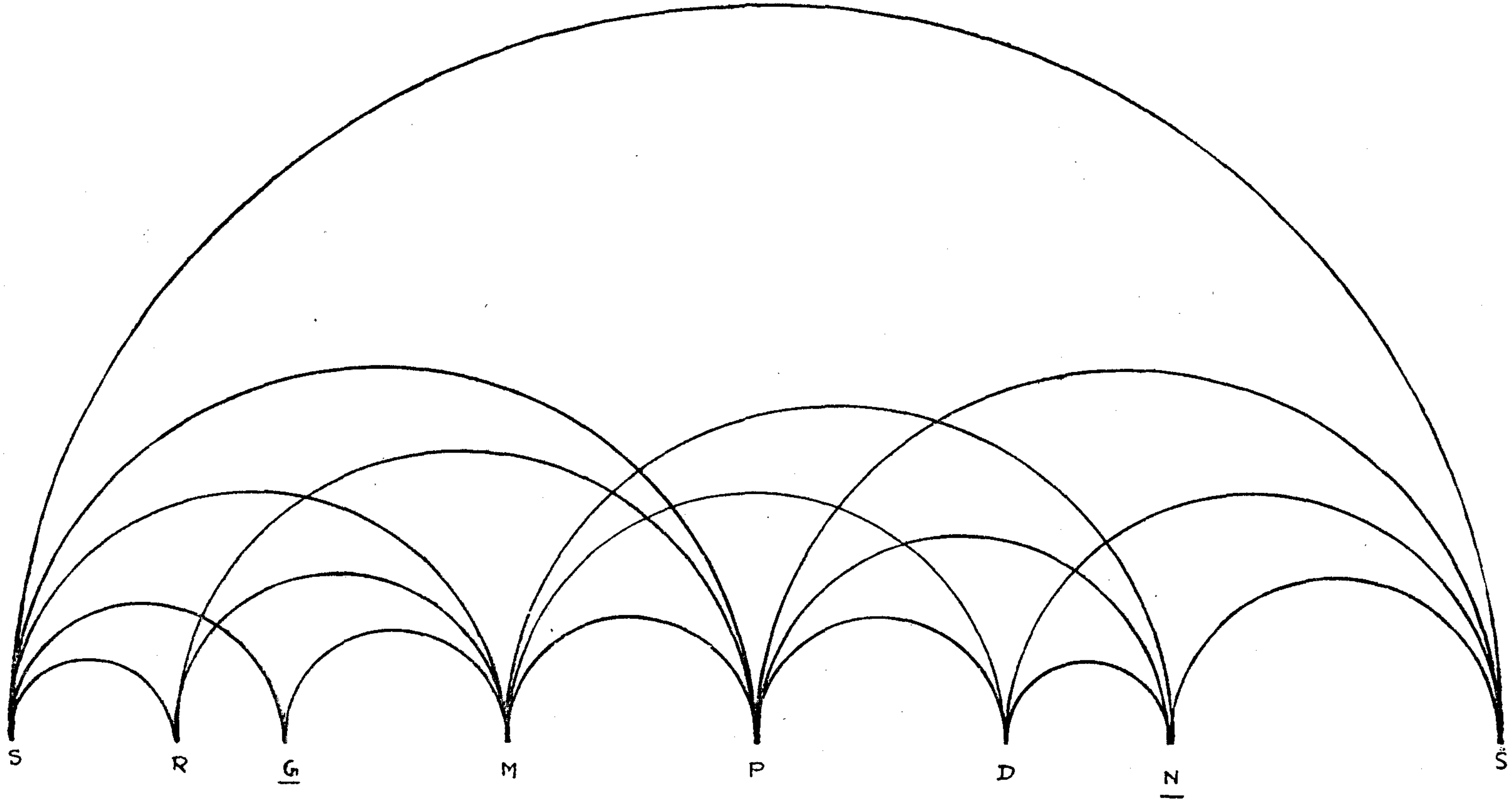

Fig. 5. The basic melodic arcs of Bharata's shadja grama. (Kafi or Kharaharapriya)

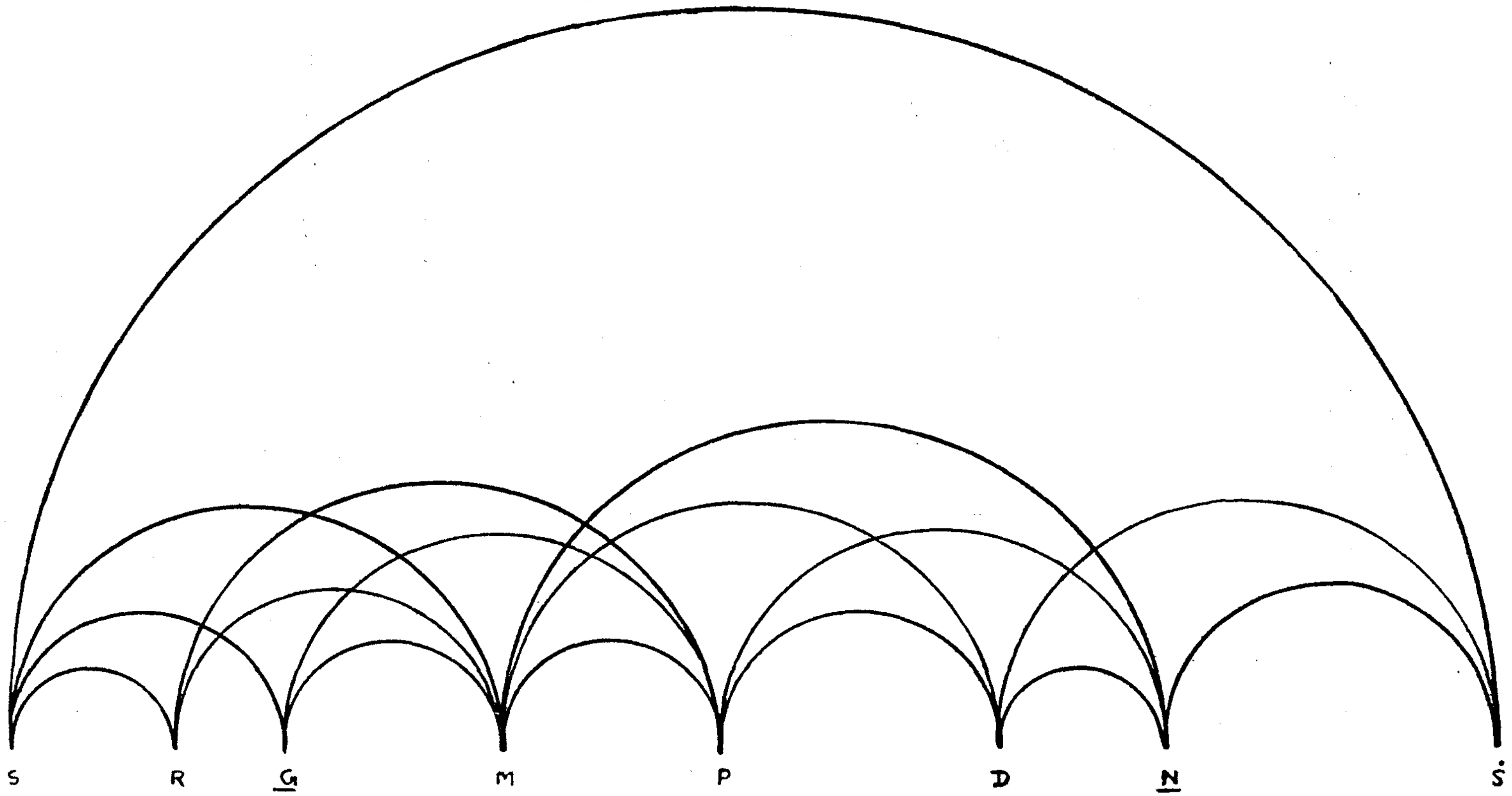

Fig. 6. The basic melodic arcs of Bharata's madhyama grama. (Kafi or Kharaharapriya)

Notes and References

* Musicometry may be said to be that branch of musicology which treats of tonal magnitude and its relations in mathematical terms and which seeks to classify and generalise the multitude of melodic phenomena numerically.

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